RESEARCH PROJECT

A subriemannian space on a manifold M is defined by a subbundle H of the tangent bundle TM, that defines the "admissible" directions at any point of M (typically, think of a mechanical system with non-holonomic constraints). Usually, H is called the *horizontal* bundle. If we equip H with a Riemannian metric, there is a naturally associated distance d on M (the so-called cc-distance), defined as the Riemannian length of the horizontal curves on M, i.e. of the curves γ such that $\gamma'(t) \in H_{\gamma(t)}$.

In the last few years, subriemannian spaces have been largely studied in several respects, such as differential geometry, geometric measure theory, subelliptic differential equations, complex variables, optimal control theory, mathematical models in neurosciences, non-holonomic mechanics, robotics.

Among all subriemannian spaces, a prominent position is taken by the so-called Carnot groups (connected and simply connected Lie groups $\mathbb{G}$ with stratified nilpotent algebra $\mathfrak{g}$), which play versus subriemannian spaces the role played by Euclidean spaces (considered as tangent spaces) versus Riemannian manifolds. In this case, the first layer of the stratification of the algebra – that can be identified with a linear subspace of the tangent space to the group at the origin – generates, by left translations, our horizontal subbundle H. Moreover, through the exponential map, Carnot groups can be identified with the Euclidean space $\mathbb{R}^n$ endowed with a (non-commutative) group law, where $n = \dim \mathfrak{g}$.

Among Carnot groups, the simplest but, at the same time, non-trivial instance is provided by Heisenberg groups $\mathbb{H}^n$, with $n \geq 1$, and, in particular, by the first Heisenberg group $\mathbb{H}^1$.

The research that the Candidate will carry on fits in the large field of Geometric Measure Theory and Differential Geometry in sub-Riemannian spaces, and, in particular, in Carnot groups.

More precisely, let us recall that in subriemannian spaces there is a distiguishrd complex of differential forms (E_0^*, d_c), the so-called Rumin's complex. In the special instance of a Carnot group $\mathbb{G}$, roughly speaking the differential forms in (E_0^*, d_c) correspond to the homogenous complemented subalgebras of $\mathfrak{g}$, i.e. to the intrinsic linear submanifolds of $\mathbb{G}$. The crucial feature of (E_0^*, d_c) is that d_c is a differential operator of order greater than 1 (that depends on the lack of commutativity of $\mathfrak{g}$.)

The themes on which the Candidate is expect to focus her/his research are:

1) Relationship between the properties of d_c and different notions of curvature for submanifolds of a subriemannian space. In particular, her/his interest should be addressed to the following notions of curvature:

1a) The notiond yield by a localized Steiner's formula in the Heisenberg group for a ϵ-neighborhood (taken with respetc to the cc-distance) of an Euclidean C^3-smooth regular surface.

1b) The notions obtained by a Riemannian approximation technique, focusing on notion of curvature for both horizontal and vertical curves, and Fenchel-type theorem for fully horizontal closed curves.

2) Differential operators on forms associated with different curvatures. In particular the Candidate should study Monge-Ampère type equations and, possibly, Hessian equations in Carnot groups.

3) Relationship between curvatures and optimal transport in Carnot groups.

4) More generally, differential forms in metric spaces.

5) Hypoellipticity propertie of non-homogeneous differential operators of higher order in subriemannian spaces, like the sub-Laplacian associated with the intrinsic differential d_c of Rumin's complex.